Recap: where did we get to last week...

- Propositional logic:
  p's and q's and the truth tables for logical connectives
- Predicate logic:
  predicates as names for sets and relations (1-place, 2-place, 3-place...)
- Quantifiers:
  for all (∀) and *there exists* (∃) for making **generalizations**

Truth Tables

HOMEWORK!!!

$\neg(p \rightarrow \neg q)$

p	q
F	F
F	T
T	F
T	T

p	q	$\neg p$	$\neg p \rightarrow q$
F	F	T	F

$\forall p \forall q.\ \neg(p \rightarrow \neg q) \leftrightarrow p \wedge q$

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

p	q	$\neg p$	$\neg p \rightarrow q$
F	F	T	F
F	T	T	T
T	F	F	T
T	T	F	T

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Recap: where did we get to last week...

- Propositional logic:
  p's and q's and the truth tables for logical connectives
- Predicate logic:
  predicates as names for sets and relations (1-place, 2-place, 3-place...)
- Quantifiers:
  for all (∀) and *there exists* (∃) for making **generalizations**

Summary : Logical Expressions

$\forall x \, \forall y$ **chase (x, y)** $\rightarrow$ **run (x)** $\wedge$ **run (y)**

Some combination of predicates and logical connectors

plus some quantifiers to 'bind' the variables

... and that gives us enough to come back and start talking about **linguistic semantics** in detail...

Sense (semantic) relations

… logic …

- **hyponyms**
- **synonyms**
  - different words that mean the same
- **opposites**
  - different words that mean the opposite of each other

Kinds of phenomena that a theory of linguistic meaning should cover

- My brother is a bachelor
- My brother has never married.

synonymy

- The anarchist assassinated the emperor.
- The emperor is dead.

entailment

... logic ...

- My brother has just come from Rome.
- My brother has never been to Rome.

contradiction

- Rich people are rich.

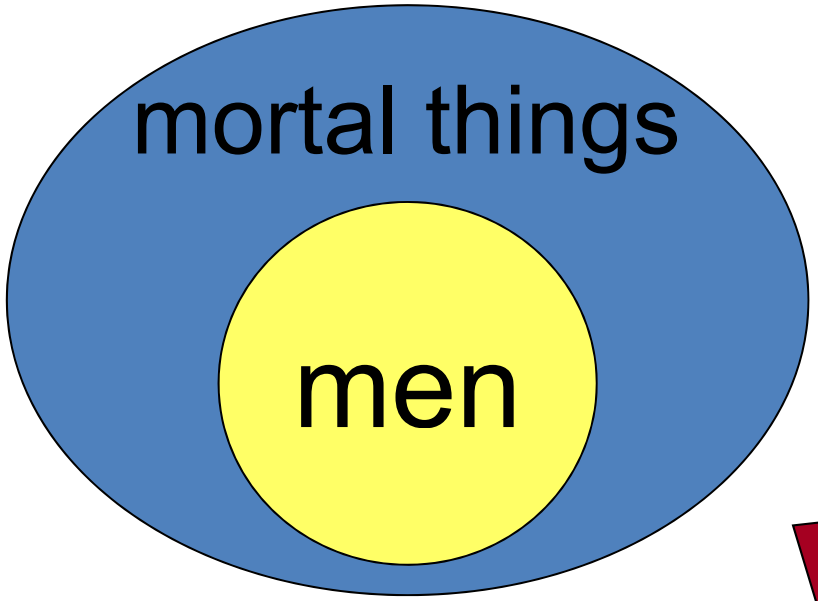
tautology

- He is a murder but he has never killed anyone.

contradiction

Using Logic

Venn diagrams

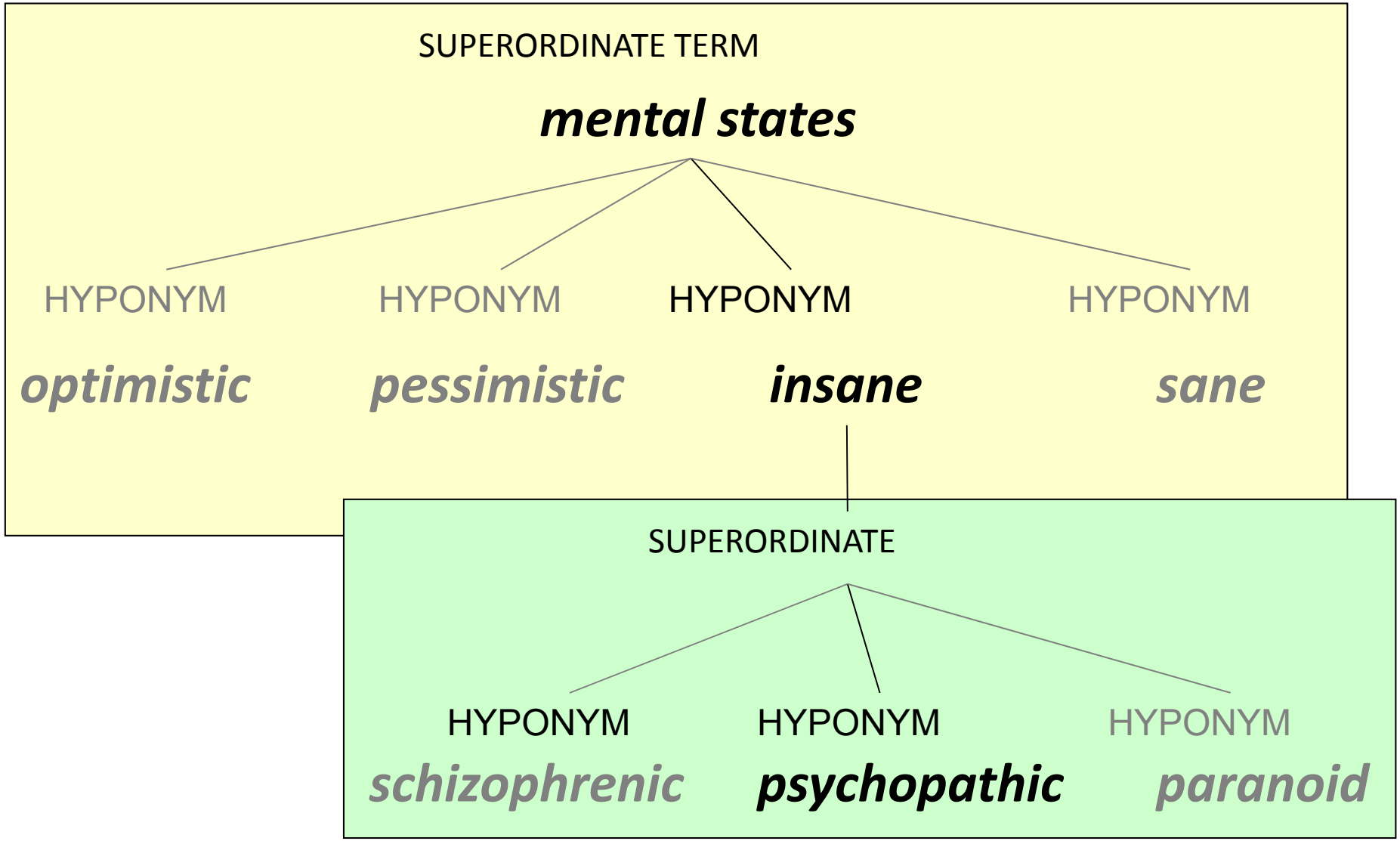


This is a statement of a subclass or subtype relationship

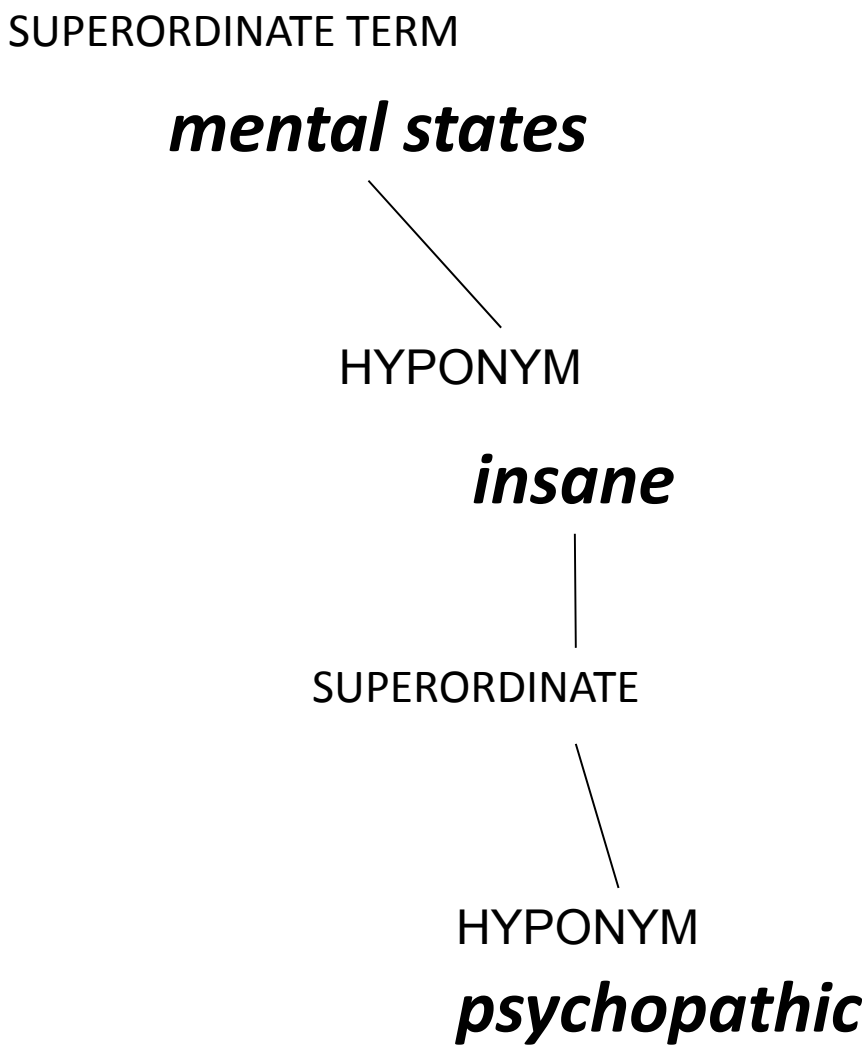
All men are mortal.

$\forall$**x: man (x)** $\rightarrow$ **mortal (x)**

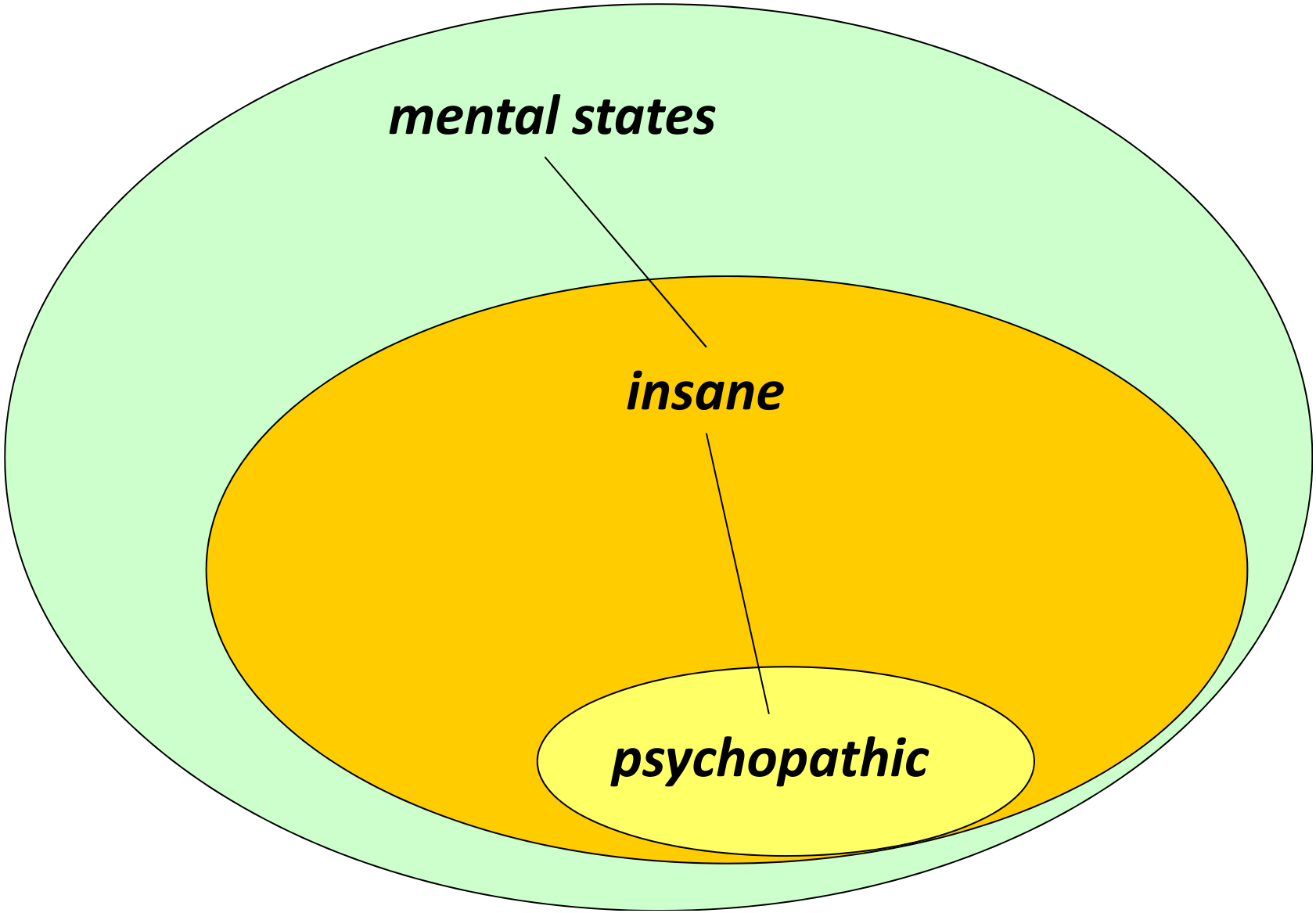
Hyponyms as **logical entailments**



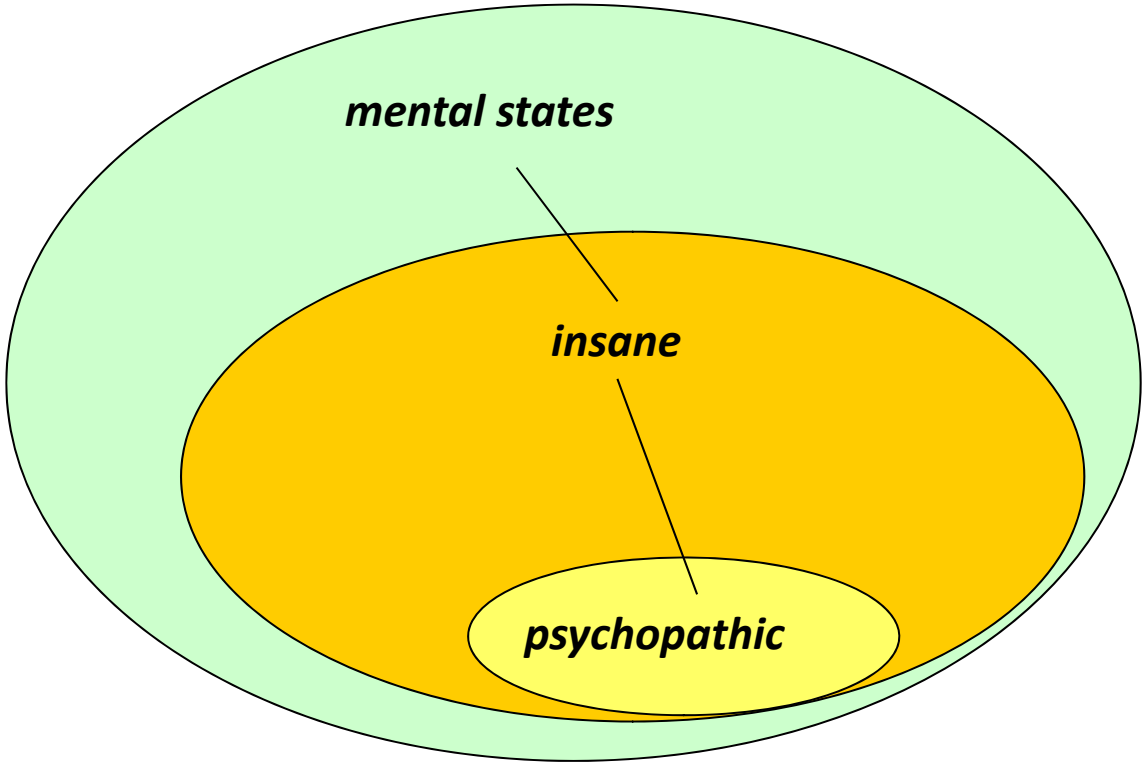
Hyponyms as logical entailments



Hyponyms as logical entailments



Hyponyms as logical entailments



All psychopaths are insane.

$\forall$**x: psychopath (x)** $\rightarrow$ **insane(x)**

Relations as **logical entailments**

- hyponyms
- **synonyms**
  - amble (x) ↔ stroll (x)

$$P(x) \leftrightarrow Q(x)$$

- **complementarity**
  - on (x) ↔ ¬ off (x)
  - off (x) ↔ ¬ on (x)

$$P(x) \leftrightarrow \neg\, Q(x)$$

- **converseness**
  - north (x, y) ↔ south (y, x)
  - parent (x, y) ↔ child (y, x)
  - wider (x, y) ↔ narrow (y, x)

$$P(x,y) \leftrightarrow Q(y,x)$$

Types of ‘difference’ in meaning

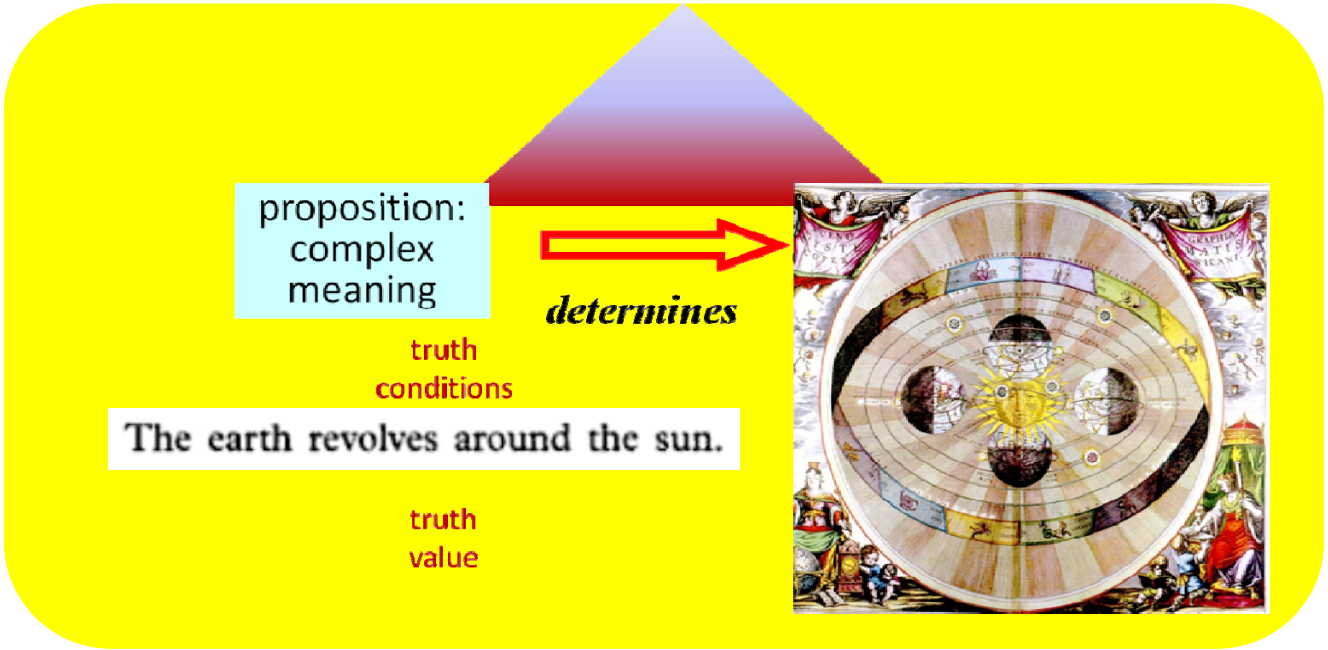
- Incompatibility: antonymy

HOMEWORK!!!

  - the water is hot *entails* the water is not cold
  - the water is cold *entails* the water is not hot
  - the water is not hot *does not entail* the water is cold
  - the water is not cold *does not entail* the water is hot

  - Gradable
    - this water is hotter than that water
    - this water is neither hot nor cold

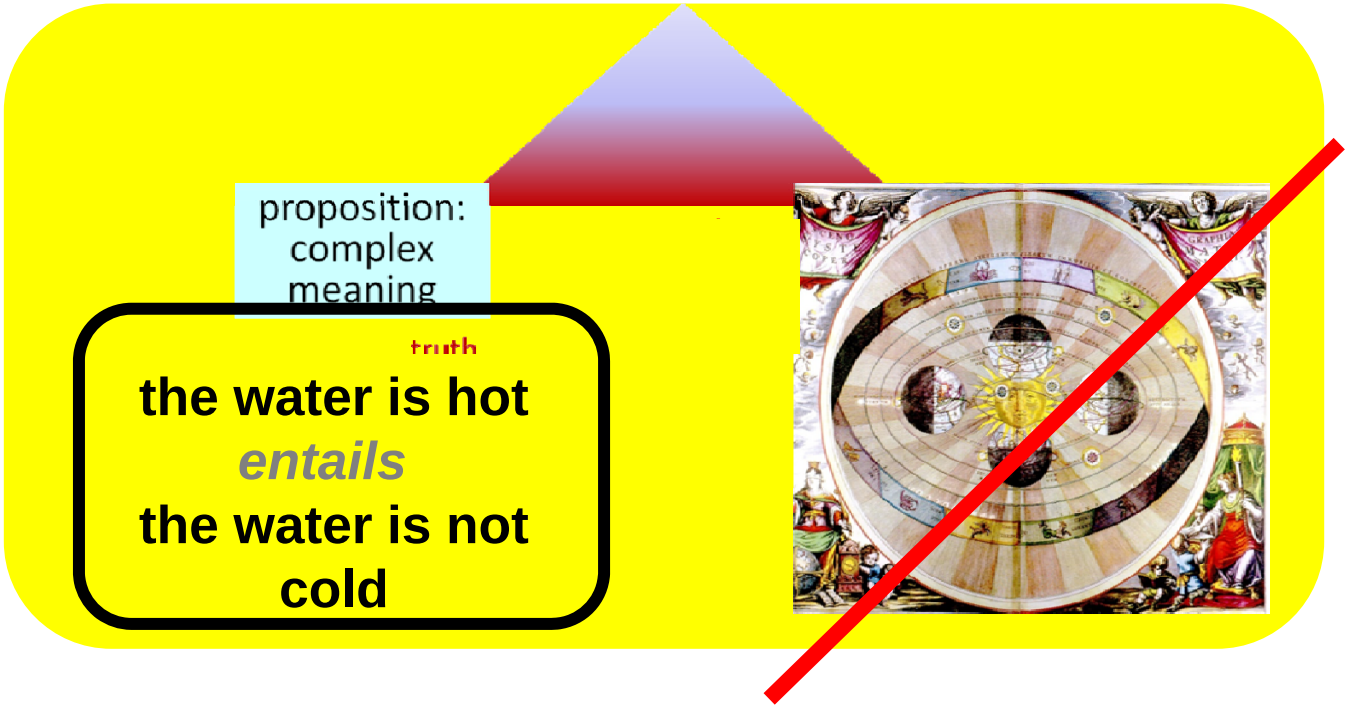
‘Analytic’ truth



Empirical / Contingent Truth

synthetic / a posteriori

‘Analytic’ truth

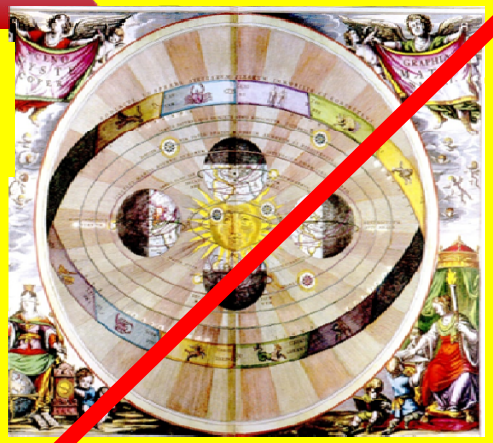


we do ***not*** have to ‘look’ at the world to
know whether this statement is true or not;
it is true by virtue of the meaning of the lexical items

‘Analytic’ truth

proposition:
complex
meaning

$\forall p \forall q.$
$\neg (p \rightarrow \neg q) \leftrightarrow p \wedge q$



again, we do ***not*** have to ‘look’ at any particular cases to know if it is true, we have **proved** that it is always true

‘Analytic’ truth

it is not the case that if I go swimming then I will not get wet

$$\dfrac{\dfrac{\underline{\text{I go swimming}}}{p} \qquad \dfrac{\underline{\text{I will not get wet}}}{\neg q}}{p \rightarrow \neg q}}{\neg}$$

$$\forall p \forall q. \; \neg (p \rightarrow \neg q) \leftrightarrow p \wedge q$$

$$\dfrac{\underline{\text{I go swimming}} \text{ and I } \underline{\text{get wet}}}{p \qquad q}$$

‘Analytic’ truth

it is not the case that if I go swimming then I will not get wet

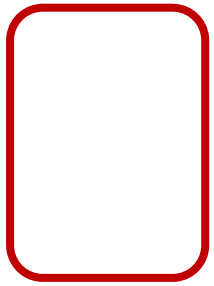
$$\neg(\underbrace{swim(I)}_{p} \rightarrow \underbrace{\neg wet(I)}_{\neg q}) \leftrightarrow \underbrace{swim(I)}_{p} \wedge \underbrace{wet(I)}_{q}$$

I go swimming and I get wet

Tautologies and Contradictions

Either **p** or not-**p**

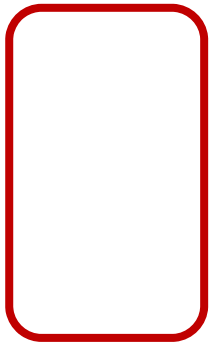
p
F
T



Tautologies and Contradictions

$\neg (p \vee \neg p)$

p
F
T



‘Entailment’

That a below **entails** b:

a. The anarchist assassinated the emperor.
b. The emperor is dead.

Entailment defined by truth:
A sentence **p** entails a sentence **q** when the truth of the first (**p**) guarantees the truth of the second (**q**), and the falsity of the second (**q**) guarantees the falsity of the first (**p**).

Step 1: If **p** (The anarchist assassinated the emperor) is true, is **q** (The emperor died) automatically true? Yes.
Step 2: If **q** (The emperor died) is false, is **p** (The anarchist assassinated the emperor) also false? Yes.
Step 3: Then **p** entails **q**. Note if **p** is false then we can’t say anything about **q**; it can be either true or false.

‘Entailment’

Composite truth table for entailment

p		q
T	→	T
F	→	T or F
F	←	F
T or F	←	T

Let's 'prove' this 'composite' truth table ...

1. What is the **logical statement** of 'entailment' as described here?

> Entailment defined by truth:
> A sentence **p** entails a sentence **q** when the truth of the first (**p**) guarantees the truth of the second (**q**), and the falsity of the second (**q**) guarantees the falsity of the first (**p**).

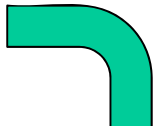
entails (p,q) iff

$$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$$

Entailment

2. What is the truth table of ‘entailment’?

$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$

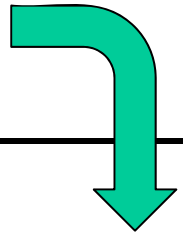


p	q	
F	F	
F	T	
T	F	
T	T	

Entailment

2. What is the truth table of ‘entailment’?

$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$



p	**q**	**$p \rightarrow q$**	**$\neg q$**	**$\neg p$**	**$\neg q \rightarrow \neg p$**	
F	F					
F	T					
T	F					
T	T					

Entailment

2. What is the truth table of ‘entailment’?

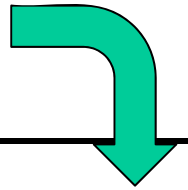
$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$

p	**q**	**$p \rightarrow q$**	**$\neg q$**	**$\neg p$**	**$\neg q \rightarrow \neg p$**	
F	F	T	T	T	T	T
F	T	T	F	T	T	T
T	F	F	T	F	F	F
T	T	T	F	F	T	T

Entailment

2. What is the truth table of ‘entailment’?

$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$

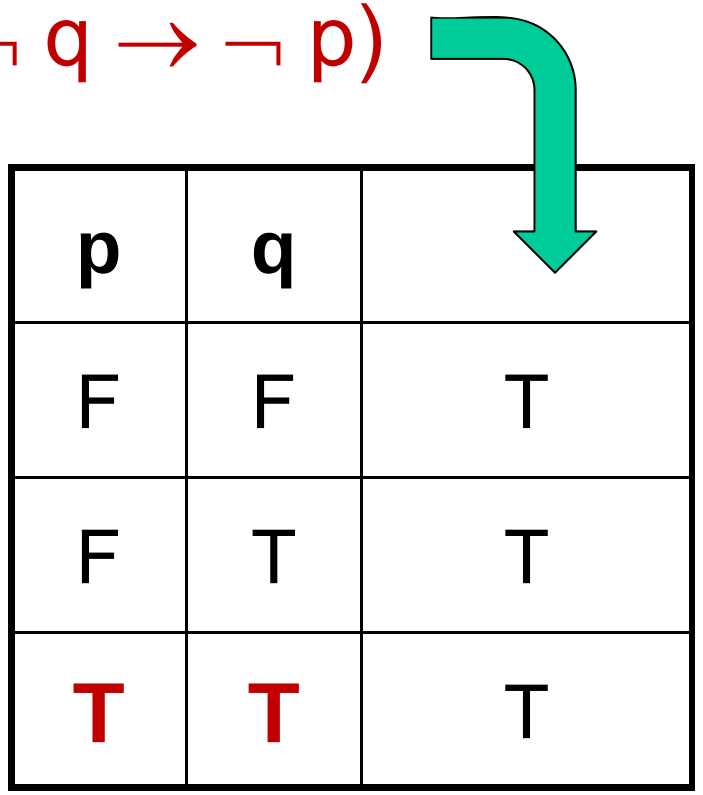


p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$	
F	F	T	T	T	T	T
F	T	T	F	T	T	T
T	F	F	T	F	F	F
T	T	T	F	F	T	T

Entailment

3. What happens when p is true?

$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$

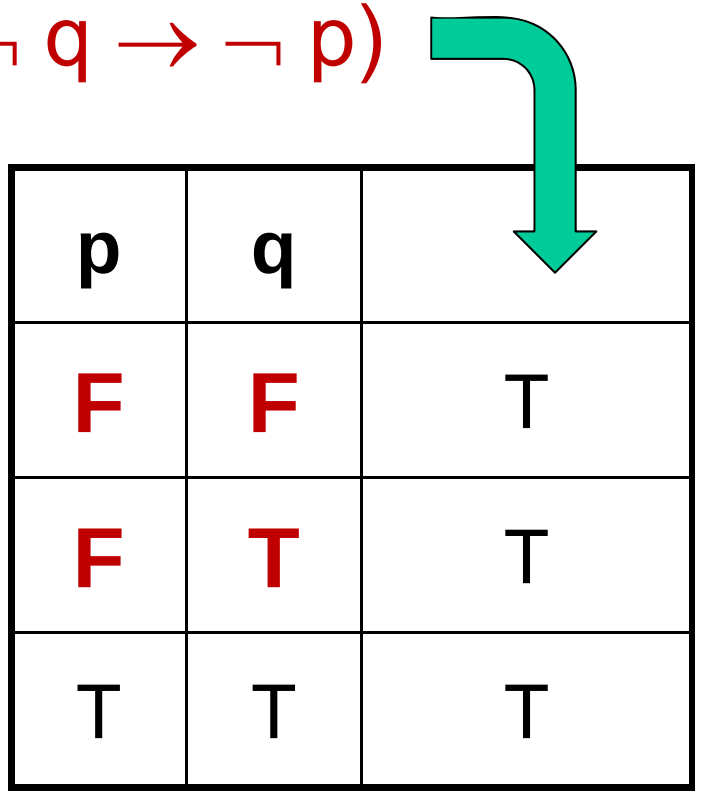


p	q	
F	F	T
F	T	T
T	**T**	T

Entailment

4. What happens when p is false?

$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$

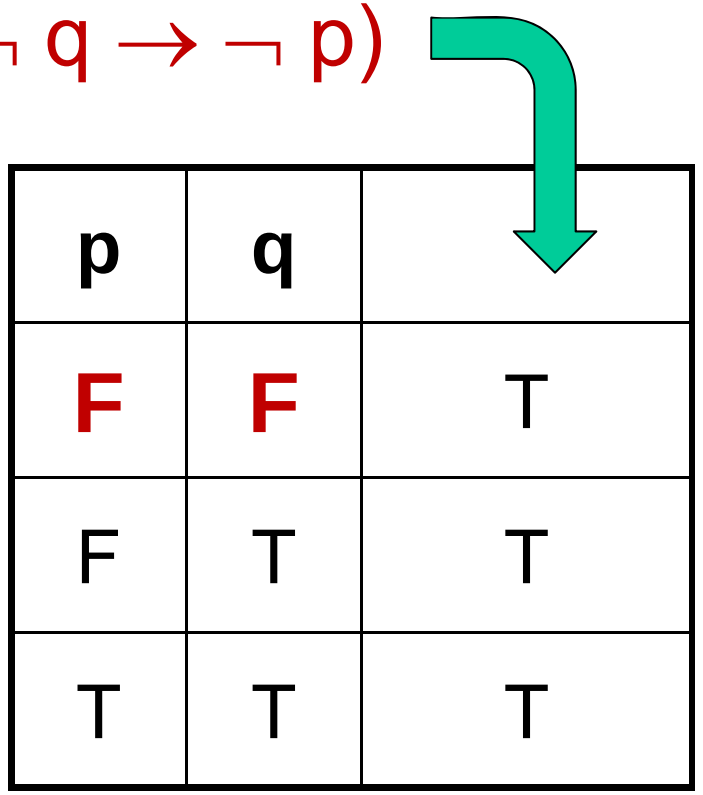


p	q	
F	**F**	T
F	**T**	T
T	T	T

Entailment

5. What happens when q is false?

$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$

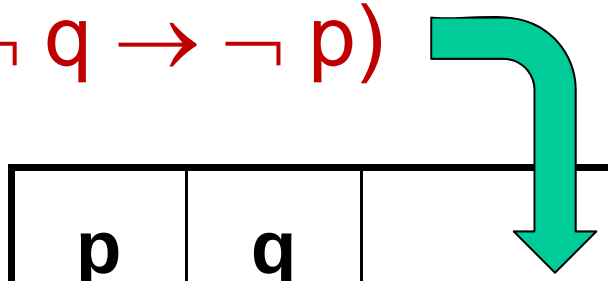


p	q	
F	**F**	T
F	T	T
T	T	T

Entailment

6. What happens when q is true?

$(p \rightarrow q) \wedge (\neg q \rightarrow \neg p)$



p	q	
F	F	T
F	**T**	T
T	**T**	T

Composite truth table for entailment

p		q
T	→	T
F	→	T or F
F	←	F
T or F	←	T

Final Point: Relation between logic and linguistic **features**

- We can also consider our **logical predicates** as **features** …

- or our **features** as **logical predicates**

semantics

Features ⇔ Logic

one-place predicates

+male	male(x)
-human	¬ human (x)
+adult	adult(x) & ¬ male(x)
-male	

The "predicate calculus"
Not, and, or

True or false

Denotational semantics

Just as we saw ages ago with…

Phonetic Features

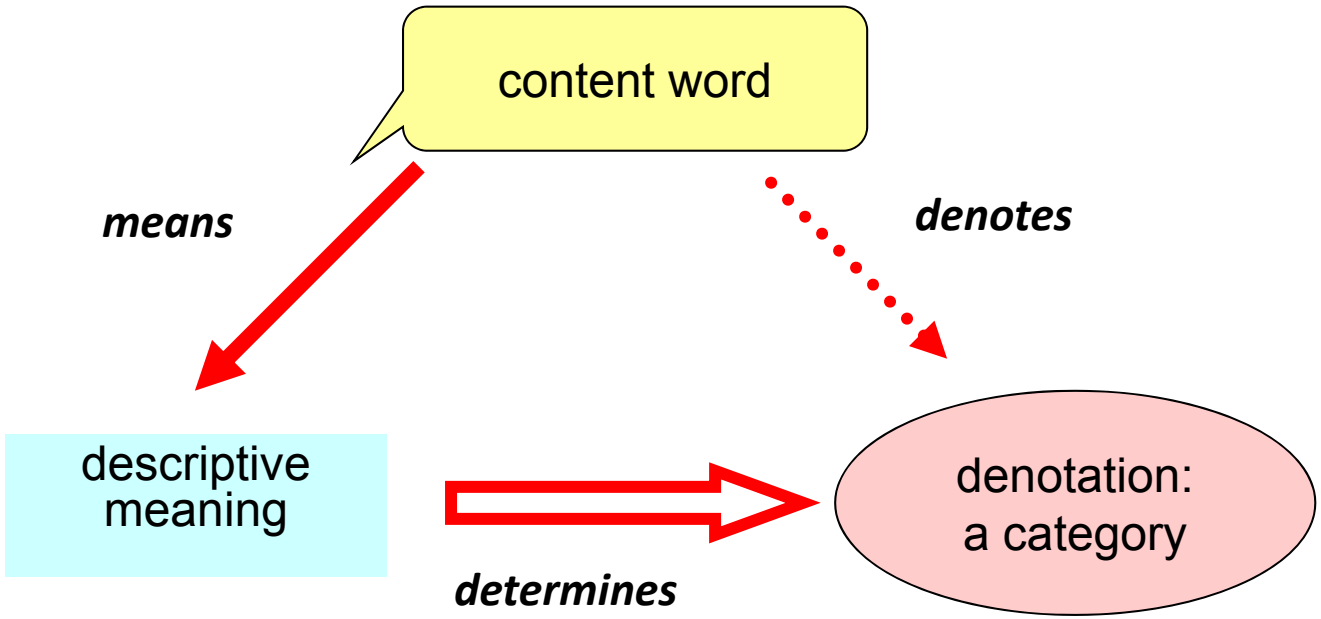
$$+ \text{nasal} \rightarrow \text{-stop}$$

$$\text{nasal (x)} \rightarrow \neg \text{ stop (x)}$$

sometimes there are ‘dependencies’ between features

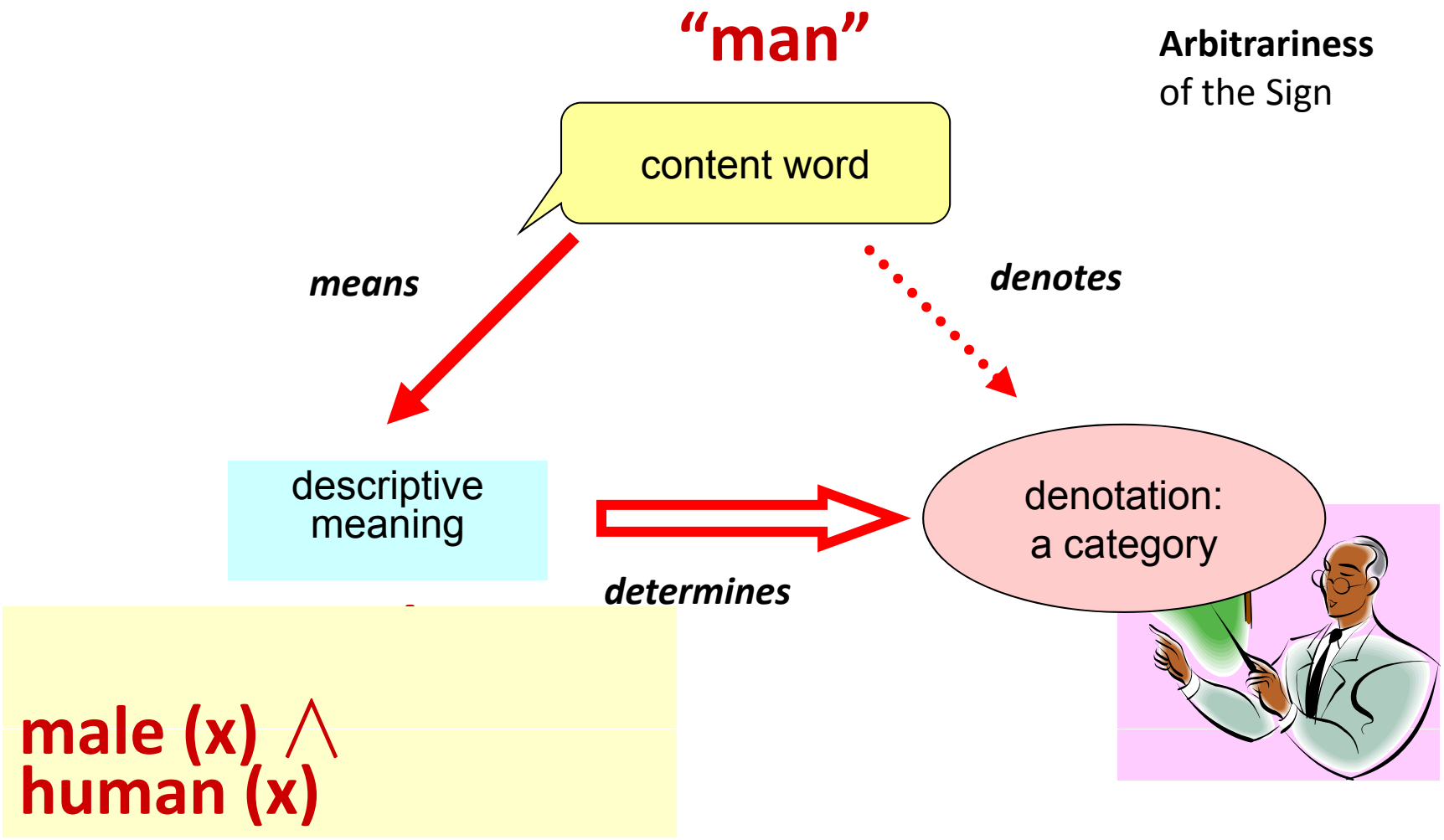
Semiotic Triangle: words

Denotational semantics



Semiotic Triangle: words

Denotational semantics

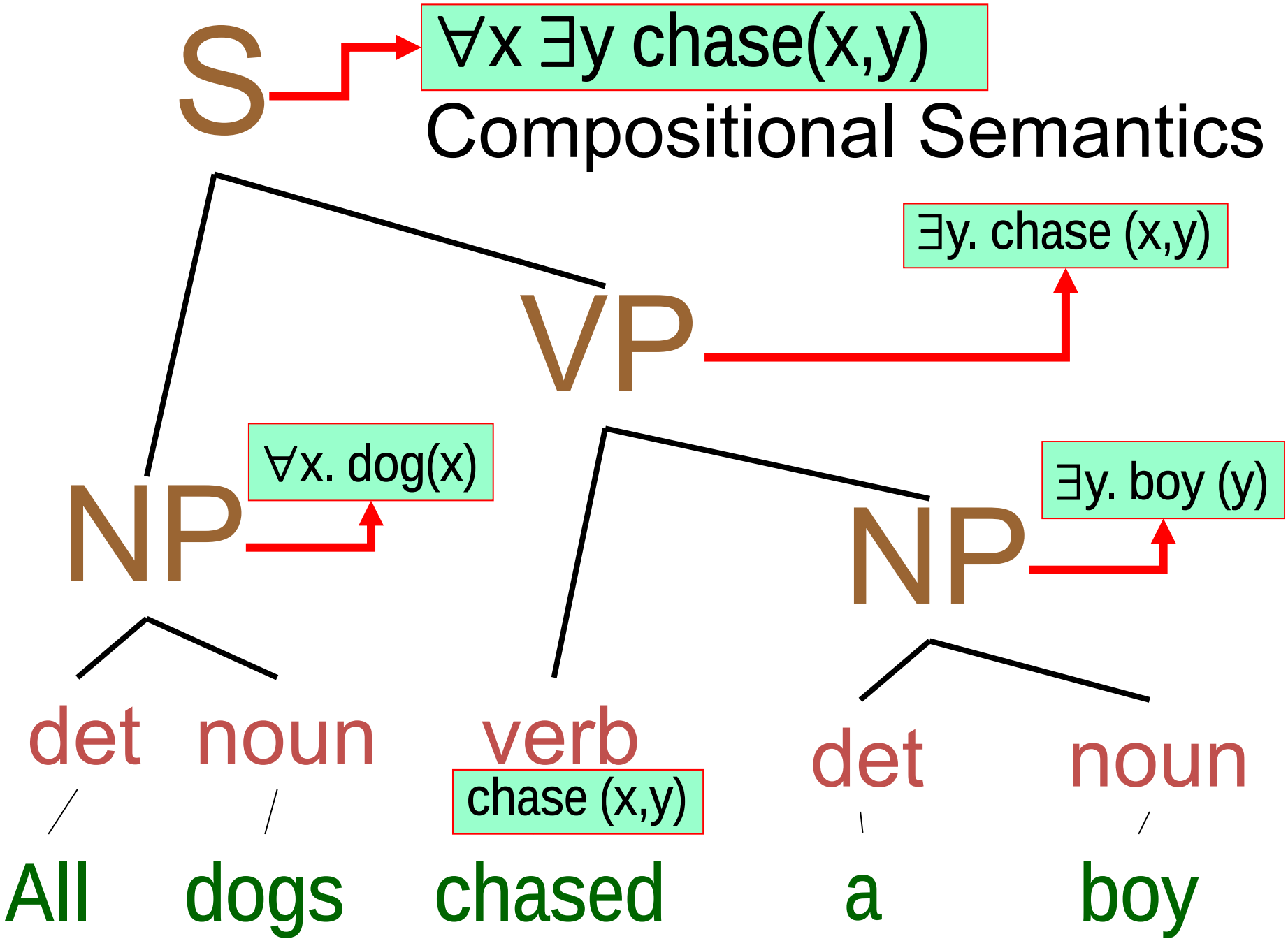


Arbitrariness of the Sign

The semiotic triangle

“rabbit”

rabbit (r) ↔
∃l ∃e ∃g .
leg(l) ∧ |l|=4 ∧
ear(e) ∧ long(e) ∧ |e|=2 ∧
grass(g) ∧ eat(r,g) ∧
run-around(r) ∧ ...


S
∀x ∃y chase(x,y)
Compositional Semantics
∃y. chase (x,y)
VP
∀x. dog(x)
NP
∃y. boy (y)
NP
det
noun
verb
chase (x,y)
det
noun
All
dogs
chased
a
boy

My brother is a bachelor

$$\exists b. \; brother(I, b) \wedge bachelor(b)$$

My brother has never married.

$$\exists b. \; brother(I, b) \wedge \neg married(b)$$

$$bachelor(x) \leftrightarrow \neg married(x)$$

Semantics rules OK!!

but needs logic to
really make it all go...