Using Logic for Linguistic Semantics

Kinds of phenomena that a theory of linguistic meaning should cover

- My brother is a bachelor
- My brother has never married.

synonymy

- The anarchist assassinated the emperor.
- The emperor is dead.

entails

- My brother has just come from Rome.
- My brother has never been to Rome.

contradicts

- Rich people are rich.

tautology

- He is a murder but he has never killed anyone.

contradiction

Sense (semantic) relations

- **hyponyms**
- **synonyms**
  - different words that mean the same
- **opposites**
  - different words that mean the opposite of each other
- **meronyms**
  - words where one thing is a part of the other

Representing these differences…

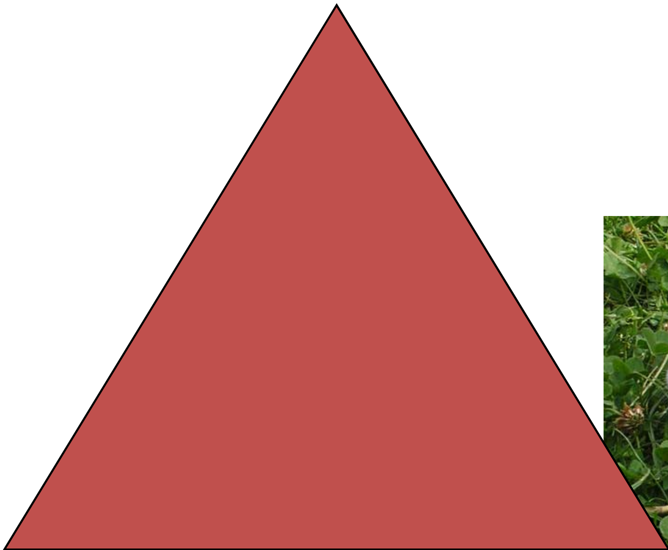
- Again, we can make everything we need much more explicit if we use….

… logic …

!!!! SO WE WILL !!!

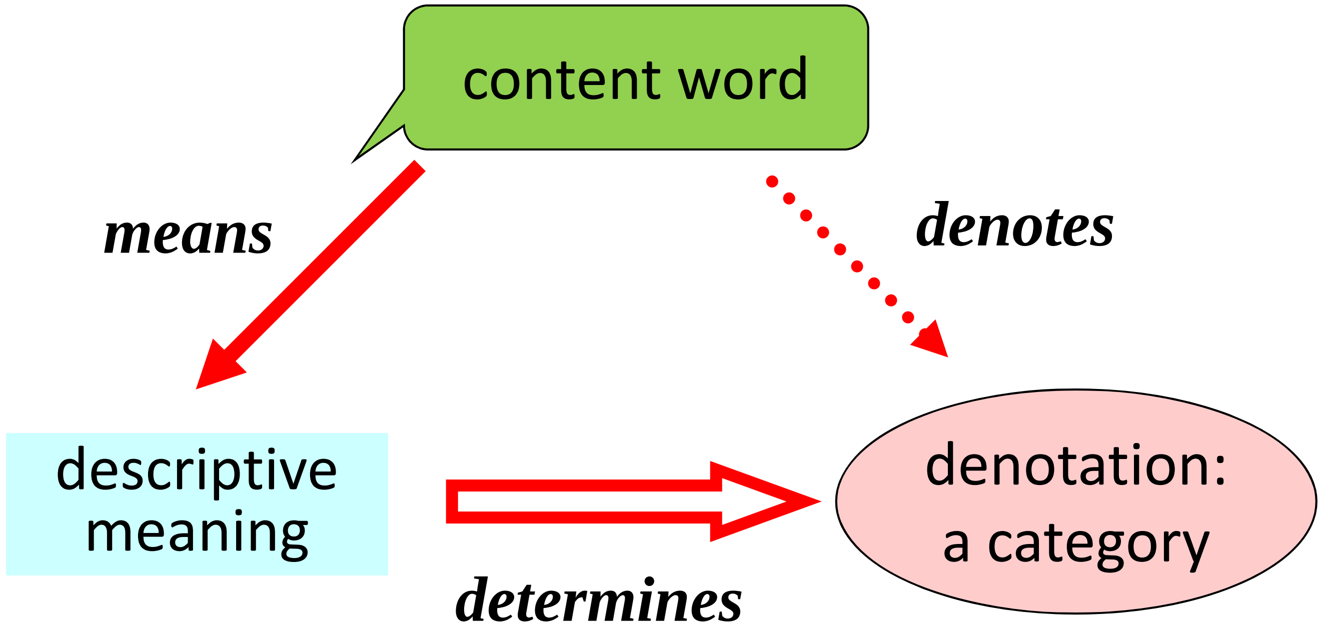
The semiotic triangle

“rabbit”

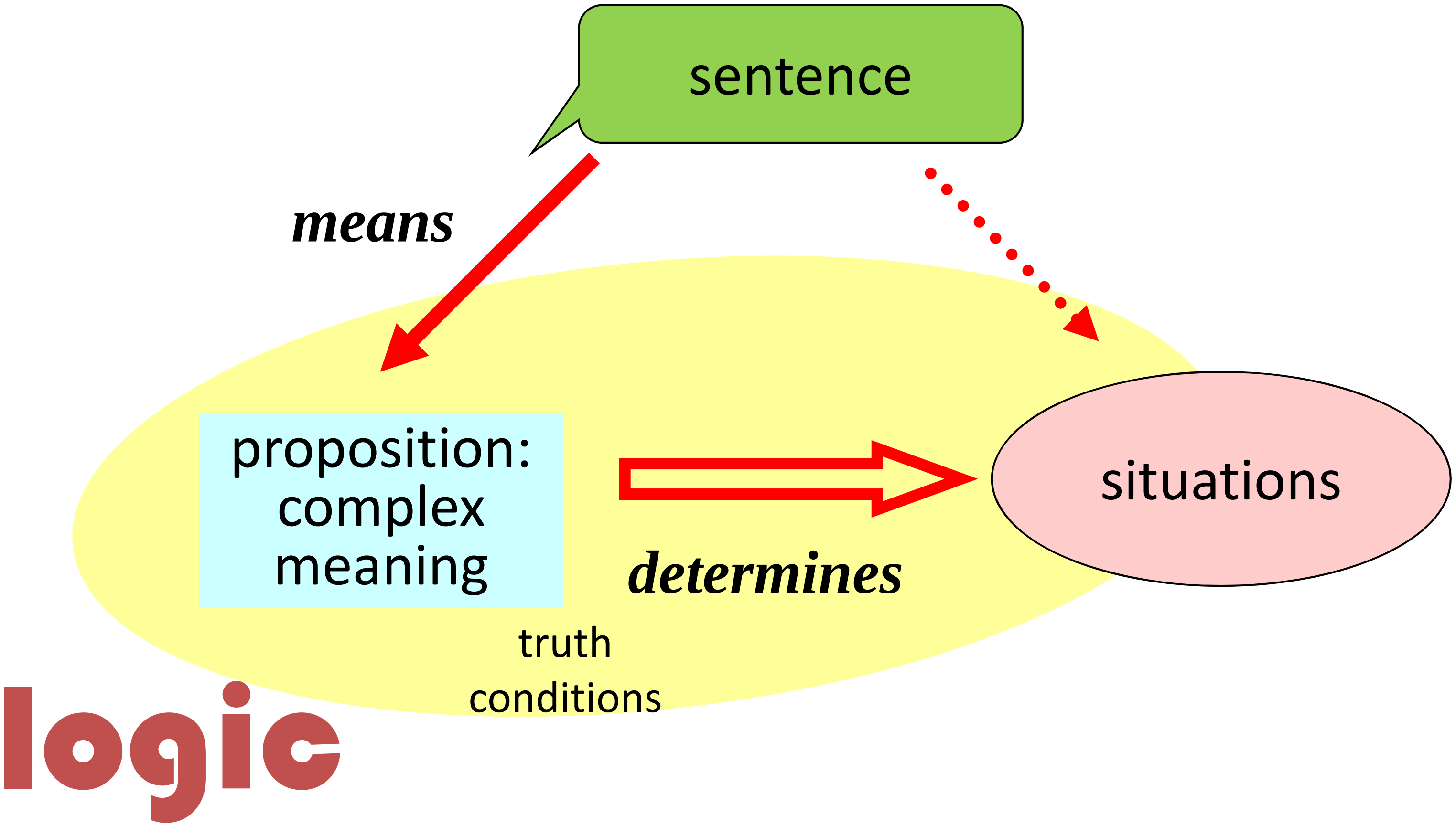


4 legged mammal
with long ears that
eats grass and hops
around
a lot …

Semiotic Triangle: words



Semiotic Triangle: sentences



Logic

- The investigation of ‘sound argument’
- Relation to Ancient Greek *rhetoric* (e.g., Aristotle)

- **What patterns of argument can be guaranteed to lead to correct conclusions?**

- One Example:

Modus Ponens

Modus Ponens

a. If Arnd left work early, then he is in the pub.
b. Arnd left work early.
c. Arnd is in the pub.

premises

conclusion

Modus Tollens

a. If Arnd has arrived, then he is in the pub. **premises**
b. Arnd is not in the pub.
c. Arnd has not arrived. **conclusion**

Hypothetical Syllogism

a. If Arnd is in the pub, then he is drinking beer.
b. If Arnd is drinking beer, then he is drinking Guinness.
c. If Arnd is in the pub, then he is drinking Guinness.

Disjunctive Syllogism

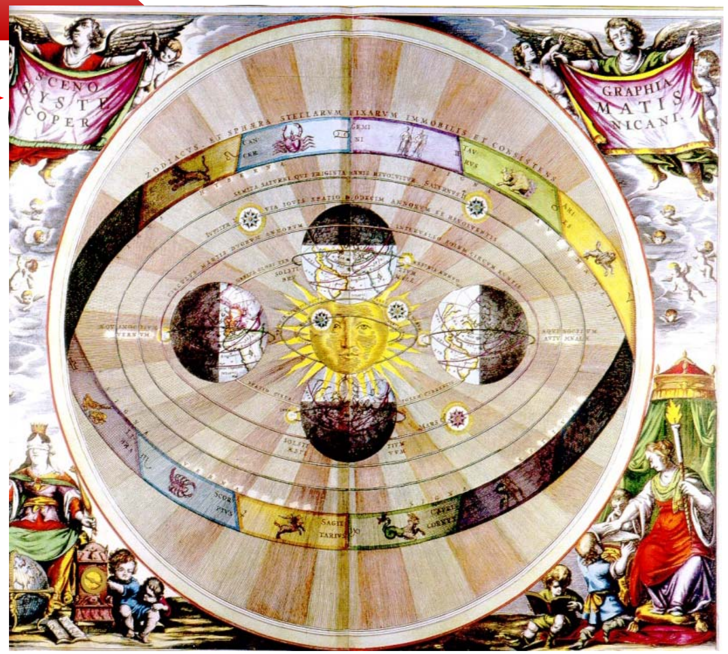
a. Arnd is in the public bar or he is in the lounge.
b. Arnd isn't in the public bar.
c. Arnd is in the lounge.

Empirical / Contingent Truth:

a **proposition** can be **true** or **false**

proposition: complex meaning

determines



truth conditions

The earth revolves around the sun.

truth value

Propositional Logic

- propositions are abbreviated by **p**, **q**, **r**, etc.
- and special logical operations are defined over those propositions (**connectives**):
  - negation (not)
  - conjunction (and)
  - disjunction (inclusive or)
  - material implication
  - biconditional implication
- using these, we can describe the ***patterns*** of argument rather than individual arguments

Modus Ponens

a. If Arnd left work early, then he is in the pub.
b. Arnd left work early.
c. Arnd is in the pub.

premises

conclusion

$$\frac{\begin{array}{l} p \rightarrow q \\ p \end{array}}{q}$$

Modus Tollens

a. If Arnd has arrived, then he is in the pub. **premises**

b. Arnd is not in the pub.

c. Arnd has not arrived. **conclusion**

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \neg p \end{array}$$

Hypothetical Syllogism

a. If Arnd is in the pub, then he is drinking beer.
b. If Arnd is drinking beer, then he is drinking Guinness.
c. If Arnd is in the pub, then he is drinking Guinness.

$$\frac{\begin{array}{l} p \rightarrow q \\ q \rightarrow r \end{array}}{p \rightarrow r}$$

Disjunctive Syllogism

a. Arnd is in the public bar or he is in the lounge.
b. Arnd isn't in the public bar.
c. Arnd is in the lounge.

$$\frac{\begin{array}{l} p \vee q \\ \neg p \end{array}}{q}$$

Truth Tables

- We also need to describe the meaning of these ‘connectives’
- Fortunately, this is very simple, because we only have propositions that can be **True** or **False**

p
T
F

Truth Tables

p ∧ q

p	q
T	T
T	F
F	T
F	F

"conjunction" / logical and

Truth Tables

p* ∨ *q

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

"disjunction" / logical or

p	q	$p \vee_e q$
T	T	F
T	F	T
F	T	T
F	F	F

exclusive or

Truth Tables

p → q

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"implication"

p is a sufficient condition for q

(p is enough to cause q, but other things might do too)

Truth Tables

$p \leftrightarrow q$ $p \equiv q$

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

"biconditional"

p is a necessary condition for q

(if q happens, p is guaranteed to have happened too)

"p if and only if q" ~ "p iff q"

Truth Tables

$\neg p \rightarrow q$

p	q		$\neg p \rightarrow q$

Truth Tables

$\neg p \to q$

p	q	$\neg p$	$\neg p \to q$
F	F	T	F
F	T	T	T
T	F	F	T
T	T	F	T

Proving logical statements

$p \vee q \leftrightarrow p \wedge q$

p ↔ q

p	q	
F	F	
F	T	
T	F	
T	T	

Truth Tables

$$\neg(p \rightarrow \neg q)$$

HOMEWORK!!!

The Language of Logic

- The investigation of 'sound argument'
- Relation to Ancient Greek rhetoric

- What patterns of argument can be guaranteed to lead to correct conclusions?

Connectives

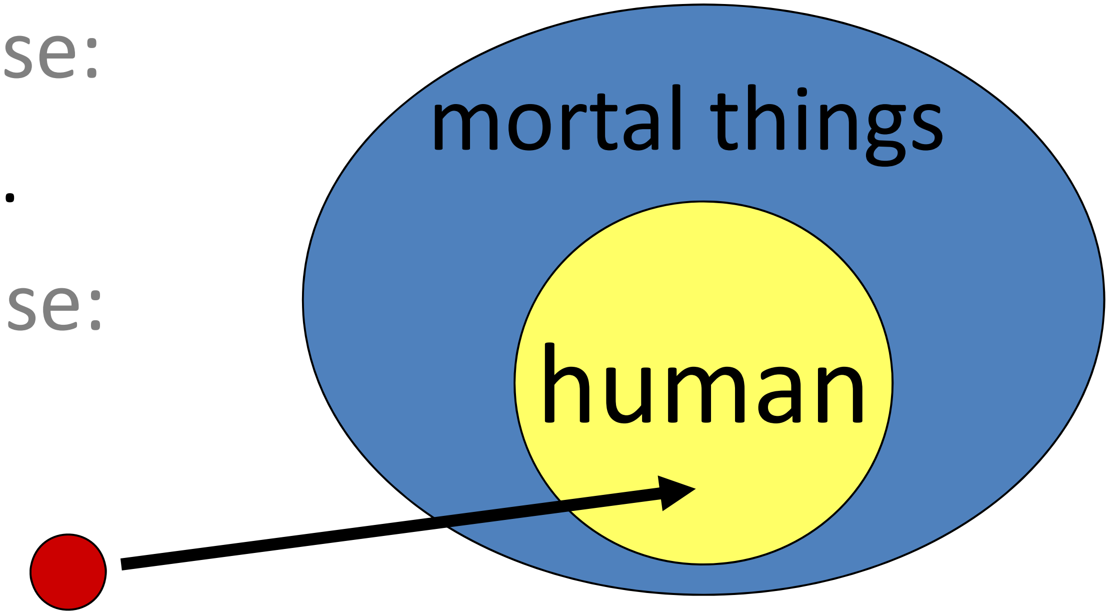
'and' : $\wedge$ 'or': $\vee$ 'not': $\neg$ 'implies' $\rightarrow$

The syllogism revisited

- Major premise:
  - All humans are mortal.
- Minor premise:
  - Socrates is human.
- Conclusion:
  - Socrates is mortal.

The syllogism revisited

- Major premise:
  - All H are M.
- Minor premise:
  - s is H.
- Conclusion:
  - s is M.



The Language of **Predicate Logic**

– Human and Mortal are
  - classes or sets

– Socrates is an
  - individual

H (s) *or* Hs

The Language of Predicate Logic

Predicates

- **“one place”**
  - door (x)
  - accountant (x)
  - book (x)
  - human (x)
  - mortal (x)

The Language of **Predicate Logic**

- Major premise:
  - All H are M.
- Minor premise:
  - s is H.
- Conclusion:
  - s is M.

H (s) *or* Hs

M (s) *or* Ms

The syllogism

- Major premise:
  - All humans are mortal.

$$Hx \rightarrow Mx$$

- Minor premise:
  - Socrates is human.

$$Hs$$

- Conclusion:
  - Socrates is mortal.

$$\frac{\quad}{Ms}$$

Logical formulae

- what about events and actions?

  - Socrates runs
  - Aristotle chases Socrates
  - The gods gave Aristotle a good idea

Predicate Logic

Predicates

- **“one place”**
  - door (x)
  - accountant (x)
  - book (x)
  - run (x)
- **“two place”**
  - eat (x, y)
  - chase (x, y)
  - read (x, y)
- **“three place”**
  - give (x, y, z)

Logical formulae

- what about events and actions?
  - Socrates runs
  - Aristotle chases Socrates
  - The gods gave Aristotle a good idea

runs (Socrates)

Logical formulae

- what about events and actions?
  - Socrates runs
  - Aristotle chases Socrates
  - The gods gave Aristotle a good idea

chase (Aristotle, Socrates)

Logical formulae

- what about events and actions?
  - Socrates runs
  - Aristotle chases Socrates
  - The gods gave Aristotle a good idea

give (Gods, Aristotle, Idea)

Logical formulae

- what about events and actions?
  - The gods gave Aristotle a good idea

a: Aristotle

Gods (g) $\wedge$
Idea (i) $\wedge$
Good (i) $\wedge$
give (g, a, i)

Finally…

- we need to put something in to keep all these ‘x’ and ‘y’ under control!
- can’t have them just running around in our formulae…

Logic

Quantifiers

- **existence:** $\exists$
- **for all:** $\forall$

All men are mortal.
Socrates is a man.
Therefore Socrates is mortal.

Logic

Quantifiers

- **existence:** $\exists$
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Logic

Quantifiers

- **existence:** $\exists$
- **for all:** $\forall$

All men are mortal.

Socrates is a man.

Therefore Socrates is mortal.

- **$\forall$x: man (x) $\rightarrow$ mortal (x)**
- **man (Socrates)**

$\rightarrow$ **mortal (Socrates)**

Logic

Quantifiers

- **existence:** $\exists$
- **for all:** $\forall$

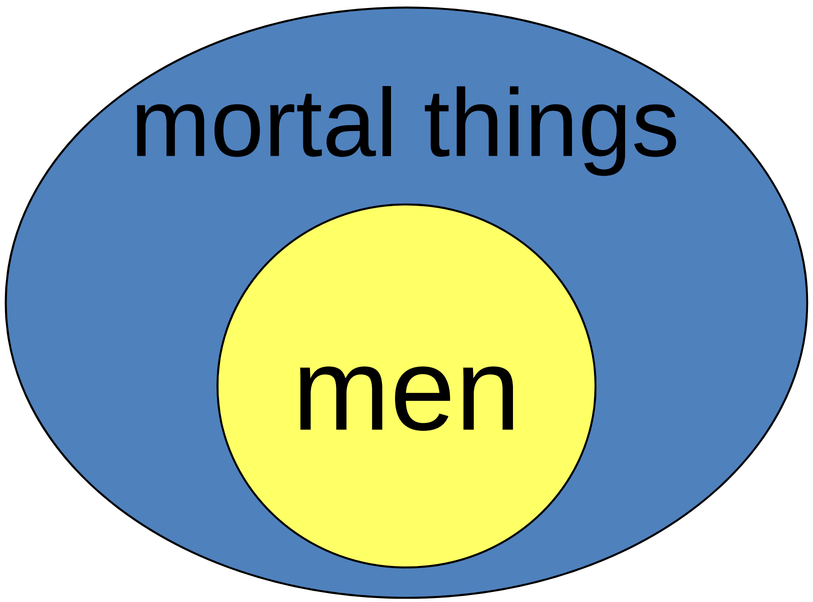
All men are mortal.

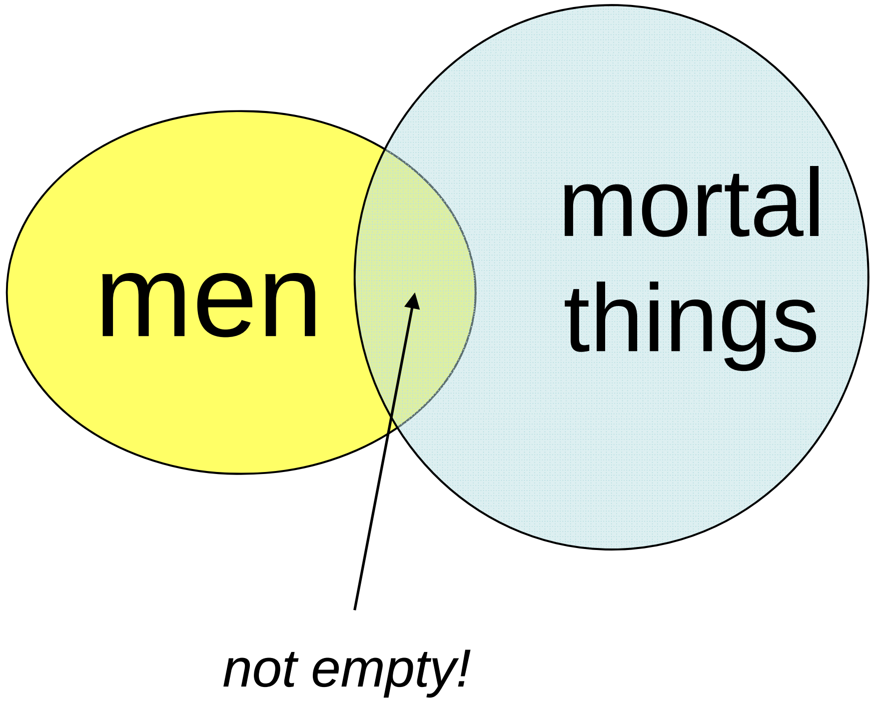
Some man is mortal.

- $\forall$**x: man (x)** $\rightarrow$ **mortal (x)**
- $\exists$ **x: man (x)** $\wedge$ **mortal (x)**

Using Logic

Venn diagrams





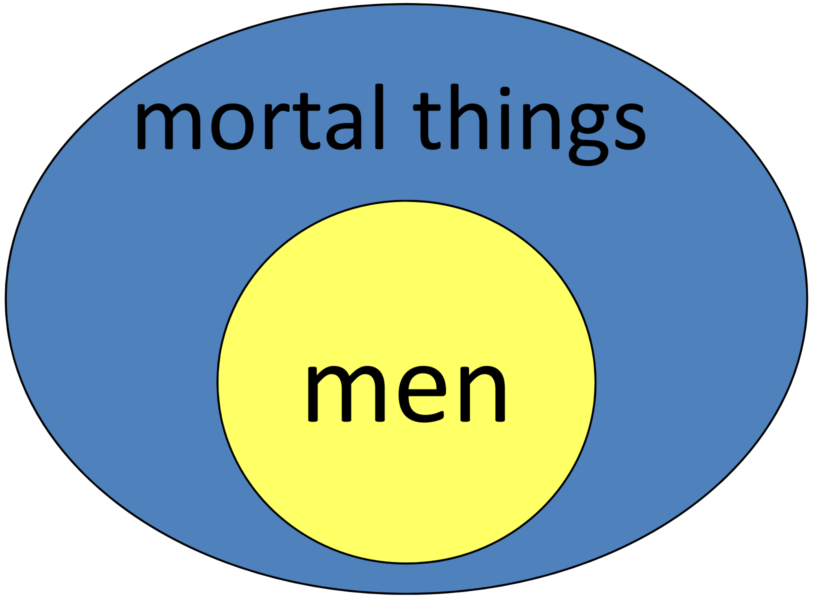
All men are mortal.

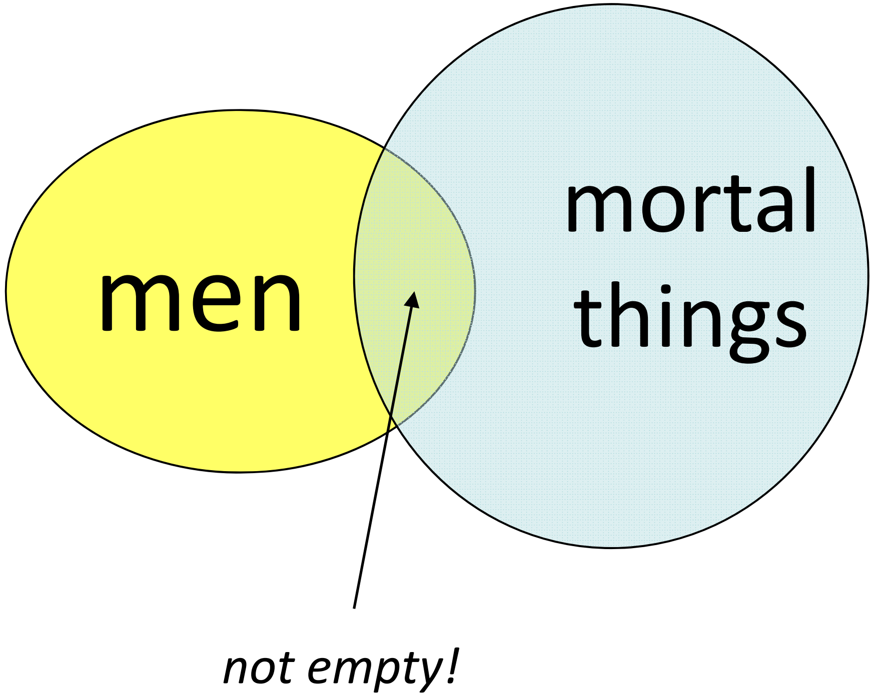
$\forall$**x: man (x)** $\rightarrow$ **mortal (x)**

Some man is mortal

$\exists$ **x: man (x)** $\wedge$ **mortal (x)**

Logic: Venn diagrams





All men are mortal.

∀x: man (x) → mortal (x)

Some man is mortal

∃ x: man (x) ∧ mortal (x)

Summary : Logical Expressions

$$\forall x \; \forall y \;\; \text{chase}(x, y) \rightarrow \text{run}(x) \wedge \text{run}(y)$$

Some combination of predicates and logical connectors

plus some quantifiers to 'bind' the variables

... and that gives us enough to come back and start talking about **linguistic semantics** in detail...