Wolfgang Wildgen

Mathematics and grammar: A contribution to the epistemology of linguistics

Lecture at the National Institute of Advanced Studies, Indian Institute of Science Campus, Bangalore, India

(17th of March 2009; 14:00)

Contents

Mathematics and grammar in the history of linguistics

- Since antiquity a certain parallelism between the alphabet, its combinatory power, and arithmetic formalisms has been evident. Thus the Indian scholars who investigated the poetic meters of Vedic chants studied the syllable structure and the combinatorial pattern of short and long syllables given a line of n syllables (cf. the classical work Chandahśāstra by Pingula; 200 BC).
- In the same period (exact dates are not know) Panini concentrated the grammatical knowledge on Sanskrit in a grammar format consisting in a list of very short rules (almost 4000), which was so well organized that it was used for centuries as the canon of linguistic knowledge and was so explicit that it allowed the great advances of comparative linguistics in the 19th century.

Two archetypes of „grammar“

- The grammar of Panini was considered an ideal of grammar writing by Bloomfield, the founding father of American linguistics, who had proposed an axiomatic system of grammatical terms in 1926 (A set of postulates for the Science of Language) and reviewed a book on Panini and Candra in 1929.
- Another more philosophical current was concerned by the logics underlying both our thinking and our language. Based on Aristotelian and Renaissance thinkers the “Grammaire générale et raisonnée” (A General and Rational Grammar) of Port Royal (cf. Arnauld and Lancelot, 1660) developed the idea that basic logical schemata underlie human rationality.

Philosophically motivated models

- Chomsky (1966) sees these authors as the precursors of his own "rational" grammar. The aspect of thinking is however reduced by Chomsky to hierarchical patterns (phrase structures) and transformations based on the intuition of the speaker about grammaticality and synonymy / ambiguity.
- Other followers of a rational grammar were the models proposed by Montague (1970), and Barwise &Perry (1983). The program they followed may be referred to philosophical trends in the Vienna circle, e.g., to Carnap's "Logische Syntax" (1934); i.e. they start from advanced logical calculi (e.g. intensional logics).
- The family of "cognitive semantics" proposed by Fillmore, Langacker, Lakoff and Talmy link cognitive capacities (perception, motor-control, memory, imagination) systematically with grammatical features. In contraposition to Chomsky, Lakoff dismisses formal languages (including logics) as tools of grammar writing and starts from a kind of "natural logic" or from folk-theories of the mind.

Descriptive mathematical grammars (Harris)

- For Zellig Harris language phenomena contain already features like discrete, linear organization, combinational power, and restrictions of this power, which ask for a mathematical treatment. As grammar was for Harris primarily concerned with forms (phonological, morphological, syntactic, and textual), their contexts and the operations of replacement and transformation of linear order, algebra was the best choice as basic tool in "mathematical linguistics".
- Similar to Bloomfield (under the impact of behaviourist psychology) he considered meaning as not accessible to scientific methods and as a consequence a formal treatment of meaning seemed to be devoid of scientific interest.
- Chomsky has at least partially followed this formalist tradition, insofar as a semantic interpretation is only attached to the basic syntactic device.

The “morphological turn” by René Thom

- The epistemological background of René Thom is Poincaré’s philosophy of sciences and mathematics and his idea of *qualitative analysis*. This idea had come-up when it became obvious that one may write down differential equations capturing the dynamics of a large system (the universe; cf. Laplace’s formula of the world), but that it is normally not possible to solve a system of such equations with spatial and temporal dynamics. One can, however, specify major characteristics of these systems such as the type of singularities which show-up. Under special conditions one may find attractors, i.e., stable states to which many process lines converge. This led to theories about *structural stability*. Many basic systems, cf. the damped pendulum may be reduced to rather simple gradient dynamics with a point-attractor.

Language and morphogenesis

- Since the 60s, Thom stood in an exchange of ideas with the British biologist Waddington and he adapted the idea of a morphogenetic field to the mathematical tools he had helped to develop (he got the Fields medal for mathematics in 1958). Such morphogenetic fields are typical for embryogenesis.
- Thom expanded this biological concept to language which he understood as a collective organism (a view also prominent in 19th century grammars). In his article: "Linguistics as a morphological discipline" (1974) he even postulates that morphologic principles operative in nature (physics, chemistry, biology) become even clearer and better visible in higher organisations, e.g., language.

The catastrophe controversy

- The more immediate controversy concerned the popularised applications of Zeeman (mainly those in the domain of psychology and sociology). As many defenders of Zeeman contributed to this controversy, its results remained open.
- The epistemological position of Thom led to a controversy on the aims of science and the merits of qualitative vs. quantitative models.
- As a general consequence many of the contributions of Zeeman to physics, theoretical biology, economics, neurology were soon recognized as standard techniques of modelling or precursors of standard models, whereas his applications to psychology and sociology remained controversial.
- Thom’s very radical revitalization of “Naturphilosophie” was further developed by Petitot and redefined in the context of Husserl’s phenomenology and Kant’s critique of reason (cf. Petitot, 1992).

Other“global players“ in the field

- As a side-effect of the catastrophe controversy of 1978-1980 other schools of dynamic modelling which had also a transdisciplinary scope of application took the lead:
- Prigogine and Nicolis: Studies on Complexity in Nature. The Brussels School developed relevant models in human geography and animal communication. The mother discipline of these contributions is chemistry (Prigogine got the noble price of chemistry in 1977). Prigogine and Stengers (1987) gave an epistemological evaluation of this endeavour.
- Haken, Weidlich, Kelso a.o., developed the idea of self-organization in coordinated systems. Haken combined the catastrophe theoretic contributions with stochastic processes. The basic idea is that of “slaving parameters” which govern the behaviour of large systems with many subsystems. The mother discipline of Haken is physics (he is a specialist of laser-physics). A series of congresses applied these ideas to many disciplines including psychology and linguistics (cf. Haken und Stadler, 1989).

Morphogenetic patterns in language

We may distinguish two basic levels in the organization of language. They correspond to classical levels like phonetics/phonology versus morphology/syntax. We presume that they mirror subsequent stages in the evolution of language:

- The dynamics of perception and production of sound patterns. They could have evolved 3 to 4 million years ago with the species Australopithecus, who lived in the savannah of East-Africa and already had upright locomotion.
- The dynamics of predication and phrasal patterns. They could have evolved with the Homo erectus/ergaster (starting 2 million years ago) and would have reached completion with the species Homo sapiens. A cognitive precursor of language may be recognized in tool manufacturing and art. (2 my BP or 700 ky BP; cf. Wildgen, 2004).

Semantic composition and neurosemantics

Consider a noun, e.g., “square”; an adjective of colour, e.g., “red”; and a present participle, e.g., “moving”:

- *red moving square*
- How does the brain compose a head-noun referring to form with two satellites referring to colour and motion?
- One distinguishes three major areas for sense related information: the visual system (subdivided into the areas V1 … V5), the occipital areas, and the parietal ones. The major binding process is one of temporal synchronization of assemblies, which form wholes (gestalts) from parts, and desynchronisation, which distinguishes figure and ground. The synchronization of two perceived stimuli can be measured in the Gamma-band (30-70 Hz) and the Beta-band (15-20 Hz) of an EEG. The fronto-parietal centres select features that are then passed on to working memory and planning.

- Composition is therefore in principle characterized by the dynamic behaviour of cell assemblies in different parts of the brain, and the relevant brain-features are the synchrony and asynchrony of firing in these assemblies.
- Further research has shown that memory and top-down processes in recognition and processing show similar mechanisms. The coding of compositional effects is, therefore, rather dynamic and temporal than static and spatial.

Complexity restrictions

- The valence restriction (to 3/in special cases 4) may find a neurodynamic reformulation if the attentional bifurcations show topological restrictions comparable to those inherent in elementary catastrophes.
- Teisman (1999: 108) formulates such restrictions „*It [the binding-by-synchrony hypothesis] also provides a plausible reason for the attentional limit of around four objects that is widely observed in the perception of brief displays and in studies of visual working memory. The different firing rates that can be easily discriminated on a background of inherent noise and accidental synchronies may set a low limit to the number of objects that can be simultaneously bound.*"
- Because valence patterns lie beyond the current experimental reach of neurological experiments, the plausibility of dynamic semantics must still rely on a rough isomorphism between patterns in the real world (physical process patterns) and linguistic forms (sentences in different languages).

Are there formal universals underlying language and other symbolic forms?

- Historically several claims of the type universals ≈ mathematical laws have been observed in the realm of music and visual art. Thus musical ratios seemed to correspond to rational numbers. For some time in Greek antiquity irrational numbers like the root of 2 seemed to fall outside a canon of beauty and regularity. They are, however, omnipresent, e.g., in the hypotenuse of a square with the side length one. If we apply Pythagoras' law, it has the value $\sqrt{2}$
- $\sqrt{1+1} = \sqrt{2}$

In a critical move against the "rationality" of Pythagoras, irrational ratios were considered as aesthetic ideals. The "most irrational proportion" (it is very difficult to approximate φ by rational numbers) is the so called "golden ratio": (φ = 1.6180339887498948482...) . It was called a "divine proportion" by Luca Pacioli (1445–1517). It may be approximated by a numerical series called the Fibonacci-series (Fibonacci's *Liber Abaci* 1202)

Chomsky's formal universals of syntax

1. The linear transition rule, which with every application (transition between two states of the abstract automaton) produces a linguistic element (a morpheme or word) and thus "writes down" the sentence from left to right.
2. The phrase structure mechanism which first generates trees of categories (mothers–daughters, sisters) and then replaces pre-terminal categories (classical word-classes) with concrete morphemes and words, thus producing chains of morphemes and sentences (by projection).
3. The transformational mechanism which takes such structures as produced in (b) as input and rearranges them. In early versions the input was called *deep-structure*, the output *surface-structure* (Later D- and S-structure).

Chomsky (1957) argues that (a) is insufficient and all grammars of human languages must be built using (b) and (c).

Chomsky comparable with Thom

- In Chomsky's Minimalist Program (1995) human language capacity is seen as a perfect device with deformations due only to the adaptation to the sensory output. These deformations are what is left from Harris' idea of transformation. What remains is mainly "morphology (checking of features) and θ-theory (assignment of semantic roles)" (ibidem: 222). Semantic role configurations are , however, the equivalent of valence patterns described in catastrophe theoretical semantics (cf. Wildgen 1982)
- There is an epistemological neighbourhood between Chomsky and Thom. Instead of algebraic rules in Chomsky's view of language, topologico-dynamic archetypes are proposed by Thom. In the style of mathematical Platonicism (i.e., there is a detached world of ideas outside our realm of knowledge which contains the *true* laws of the world), Thom argues that the morphologies derivable via catastrophe theory are universal structures valid for nature and man (including man's cultural world).

How relevant is mathematics for linguistic model building?

- *Apply any mathematics you can.* In this case mathematics are considered as a universal tool for model-building independent from the discipline or phenomenon in question. The introduction to mathematical linguistics by Hubey (1999) comes close to that position. This seems to be the most neutral position, but it neglects the fact that historically geometry, arithmetics, logics, algebra, and probability theory were developed in view of a class of problems which came up in different contexts and in different domains of applications.
- *Language as a formal language.* Montague's position (see the title of his paper: "English as a formal language") is probably the best representative. When Chomsky (1957: 13) equalized language with the finite or infinite set produced by a generative grammar he took a similar view, although he very quickly rejected Markov chains and statistical dynamics, and he also never considered continuous models or methods of geometry and topology. In fact, his mathematical education encompassed only logics and basic automata theory.

- *Language contains (implicitly) mathematics.* Zellig Harris assumes that language contains its own metalanguage which is the major tool of linguists. But implicitly this natural metalanguage shares features with parts of mathematics and allows the application of these mathematics. The value of the application depends on its naturalness, i.e., on the equivalence between mathematics and the natural metalanguage we use when we speak about language. As a consequence, only a very specific choice of mathematical concepts may be applied in linguistics (cf. Harris, 1991).
- *Mathematics is applied cognitive semantics.* Finally one could argue that the cognitive content of mathematics depends on the semantics of natural language, i.e., we need an analysis of mathematics in terms of cognitive semantics, e.g., of Lakoff's theory of conceptual metaphors in order to judge the relevance of mathematics. In this case linguistic analysis may be applied to mathematics but not vice-versa (cf. Lakoff and Nunez, 2000), and the "objectivistic" paradigm in linguistics must be rejected (cf. Lakoff, 1987).

An integrative view proposed by Cassirer

- In the field of “symbolic forms” magical and mythical thinking are considered as the basic level, at which form (signifiant) and meaning (signifié) are not yet distinguished. The name of the God (or spirit) is directly related to the God (spirit); if you use the name, the God is called forward and must appear. In some religions it is therefore not allowed to spell the name of God as this would force God. Holy scriptures are interpreted as written by God (via the hand of some prophet); their form and meaning is fixed for eternity (in a sense they are not linguistic expressions in the normal sense).

- The mythical semantics are rather fluid than discrete and transitions between categories are easy as if no frontiers existed. Metamorphoses between humans, animals, plants or stones are simple. Sentences may have many different meanings related to different levels of interpretations and some of these meanings may be hidden and only accessible to priests or initiated persons, who are not allowed to teach these meanings to non initiated persons. Thus this code is closed and does only unfold in special cult-actions or in situations of trance, where a kind of communication beyond language (glossolalia or the speaking-in-tongues) appears. In a recent study one of the observed women said: “You’re not really out of control. But you have no control over what’s happening. You’re just flowing”.

- In spoken and even more in written language the separation of form and meaning is much clearer and in situations of multilingual communication the arbitrariness of the relation between form and meaning is becoming obvious and conscious for all speakers. In writing praxis the discretizations of words and the combinatorial nature of word- and sentence formation becomes accessible to consciousness. Partially it is a product of the writing technique and not primarily a feature of natural languages; this means that with the invention of writing a cultural bifurcation between spoken and written languages took place and the correspondent language awareness due to writing began to dominate linguistic theorizing.

The phase of „pure meaning“

- Finally with the rise of mathematics (Euclid may be considered as a first climax) the stage of “pure meaning” (Cassirer: “reine Bedeutung”) is reached. Ideally the arbitrariness of natural languages is overcome and a universal language is created which reflects perfectly the architecture of thinking hidden by linguistic forms. This is a dream articulated by many authors in the 17th century, when the global language of erudition, Latin, was replaced by national languages. Port Royal logics were announced as an *art of thinking* and Leibniz formulated his “Characteristica universalis” which was not only conceived as a universal tool for the expression of knowledge , it should even allow the finding of new knowledge.

Restrictions of the universality or “pureness” of logics and mathematics

1. Aristotelian logics and syllogistic and to some extent all modern logical calculi are based on generalization of existing languages, mostly Indo-European languages. This criticisms was put forward by Benjamin Lee Whorf who formulated the hypotheses of linguistic relativism. Our thinking is shaped by our language.
2. Many fields of mathematics are linked to special fields of problem solving which occurred at specific stages of cultural evolution, e.g. the measurement of land or the construction o pyramids in Egypt, the exact description of the sky and of astrological situations in Mesopotamia, the calculation of the calendar in many religions (also in Maya calendars). The historical unfolding of mathematical knowledge has therefore followed rather specific cultural developments after the Neolithic revolution.

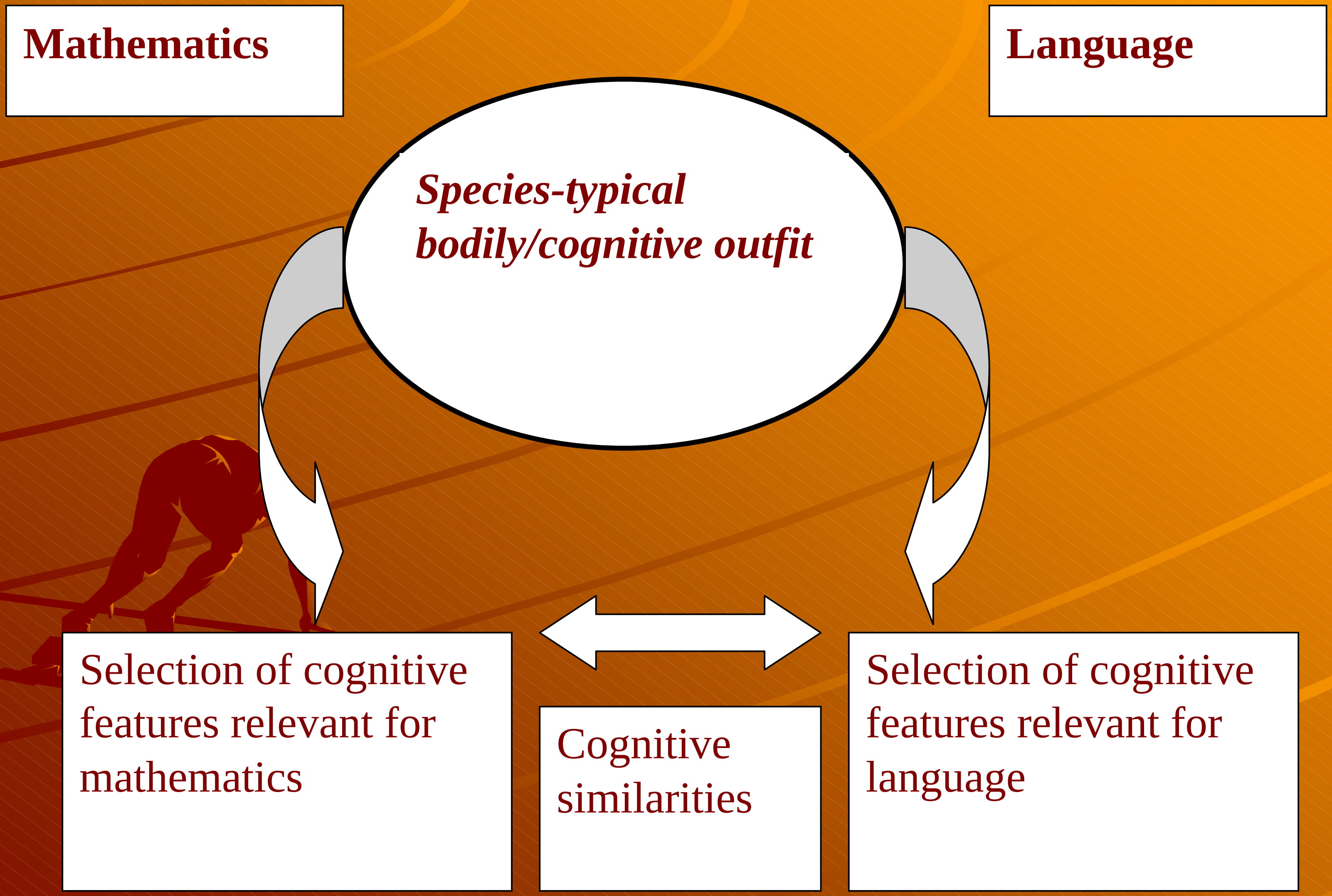
3. Mathematics (as languages) has to be congruent with human cognitive capacities; therefore they are relative to the genetic outfit of our species. Other species with a different cognitive profile will therefore have different mathematics. In this sense mathematics cannot and should not be universal. But they are neither a direct outflow of these capacities as Lakoff and Nunez (2000) make us believe. As shown in (2) the context of very specific cultural innovations triggers and shapes the development of mathematics.
4. The cognitive outfit of humans (minimal or average outfit) is itself not homogeneous because different capacities have evolved at very different evolutionary stages. Thus perceptual and motor capacities and the corresponding facilities of spatial orientation have evolved very early (after the Cambrian evolution 500 my ago), specific auditory and phonatory capacities of humans probably evolved during the last four million years and the capacities of lexical networks and quick syntactic processes could have evolved in the time before the speciation of modern humans (after 500.000 y BP).

Our hypothesis

- Insofar our cognitive capacities are a kind of evolutionary patch-work this should also be true for languages and mathematics. Our hypothesis is therefore: “Pure” meanings beyond cognitive restrictions specific for our species and beyond cultural relativity are an intellectual illusion or an utopia. This utopia has been cultivated since the 17th century and reached a climax around 1900 with Frege, the Vienna circle, the “Principia Mathematica” by Whitehead and Russell and Neurath’s program of a universal encyclopaedia of knowledge.
- If this is true the question remains, why modern mathematics were so globally efficient as a basis of modern technologies and computer sciences and thus more universal than languages.

Major differences of functionality seen in an evolutionary perspective

- Now, if lithic technologies are the baseline of the symbolic form “technology”, than this baseline is much earlier than the emergence of language and therefore more stable (or more universal, i.e. less dependent on cultural developments). Moreover, its selection is controlled by the advantage it gives for the survival of the individual and the group. This evolutionary control is not operative in the same extent in the case of language which does rather serve communicational needs and is very weekly bound to forces of survival. This allows for a great variability and a quick change, such that cultural effects tend to accumulate in languages.
- Although mathematics is a rather late cultural achievement it has strong links to technologies which are deeply rooted in human evolution and controlled by survival criteria (fitness effects). This makes them less divergent and more universal (or convergent in time).


Mathematics
Language
Species-typical bodily/cognitive outfit
Selection of cognitive features relevant for mathematics
Cognitive similarities
Selection of cognitive features relevant for language

Mathematics

Language

Cultural relativity/ specific needs in cultural contexts

Selection of technological solutions relevant for a stage of cultural evolution

Cultural similarities

Selection of communicative solutions relevant for specific societies and their coherence

Consequences in the case mathematics are applied to language

1. Both have a bodily and cognitive foundation and are restricted to the human species. The selection of sub-components of human cognitive endowment is, however, different. It is clear that emotional/sexual, poetic/expressive functions which may be crucial in language are underrepresented in mathematics. Both symbolic forms are highly abstract but mathematics specialized the linguistic abstraction in the domains of geometry and algebra/combinatory. These differences remain, however, gradual and are not absolute, but the cognitive profiles of both symbolic forms are clearly different and have become historically more and more independent from one another.

2. Both are culturally relative, but the link of early mathematics to technologies (and perhaps myth) makes them less variable. The evolutionary control of technologies is stricter (more narrow) than in the case of languages, where many different kinds of structural organization may serve the same needs for communication. Therefore if some kind of less culturally variable architecture behind languages is the problem to be solved, mathematics may be helpful in the search for universals (but they are not the archive of such universals).
3. In a different sense languages are (functionally) more universal than mathematics. Every human is able to learn one or more languages, but many humans have difficulties in learning or using mathematics. Language is the basic link of the species and it guarantees its unity.

Conclusion

- The (four) positions enumerated in the relation between mathematics and grammar mentionned earlierare insufficient. One must consider the intricate relation between the two symbolic forms: *language* and *mathematics* in order to find relevant ways to elucidate one form with the help of the other.
- The basic dilemma is, however, that language is an early acquired symbolic form which is in majority sub-conscious and its reconstruction with the help of the highly conscious mathematical tools is a dangerous operation involving the transfer from the subconscious to the conscious via a symbolic form which is cognitively and functionally different.

Some publications of the author

Wildgen, Wolfgang, 1982. Catastrophe Theoretic Semantics. An Elaboration and Application of René Thom’s Theory, Benjamins, Amsterdam/Philadelphia.

Wildgen, Wolfgang, 1994. Process, Image, and Meaning. A Realistic Model of the Meanings of Sentences and Narrative Texts, Benjamins, Amsterdam/Philadelphia.

Wildgen, Wolfgang, 2004. The Evolution of Human Languages. Scenarios, Principles, and Cultural Dynamics, Benjamins, Amsterdam/Philadelphia.

Wildgen, Wolfgang und Barend van Heusden (eds.) forthcoming 2009. Meta-representation. From Selforganization in Nature to Cultural Artefacts, Lang, Bern.

Brandt, Per Aage und Wolfgang Wildgen (eds.) forthcoming 2009. Semiosis and Catastrophes. René Thom's Semiotic Heritage, Lang, Bern.

Further literature used

- Arnauld, Antoine and Claude Lancelot (1981). Grammaire générale et raisonnée. Paris: Le Petit. Facs. Reprint. Menston : Scolar Pr, 1967.
- Bose, D.M., S.N. Sen, and B.V. Subbarayappa (1971). A Concise History of Science in India, Indian National Science Academy, New Delhi (reprinted 1989).
- Callahan, J.J. (1977). Singularities and Plane Maps II: Sketching Catastrophes, in: American Mathematical Monthly, December, 765-803.
- Chomsky, Noam (1975). The Minimalist Program, Cambridge (Mass.), MIT Press.
- Kak, Subhash (2004). The Golden Mean and the Physics of Aesthetics, Archive of Physics , 0411195. www.**cs.okstate.edu**/~subhashk/GoldenMean.pdf (27.1.2009)
- Kelso, J. A. Scott (1997): Dynamic Patterns. The Self-Organization of Brain and Behavior. Bradford: Cambridge (Mass.).
- Lakoff, G. (1987). Women, Fire and Dangerous Things: What Categories Reveal about the Mind , Chicago, University of Chicago Press.
- Lakoff, George et Rafael E. Nuñez (2000). Where Mathematics Comes from: How the Embodied Mind Brings Mathematics into Being, New York, Basic Books.

- Montague, R. (1970). « Universal Grammar », Theoria 36, 373-398.
- Petitot, Jean (1992). Physique du Sens. De la théorie des singularités aux structures sémio-narratives. Paris: Editions du CNRS.
- Petitot, Jean, 2003. Morphogenesis of Meaning (translated by Franson Manjali; a revisited version of "Morphogenèse du sens", 1985). Bern: Lang.
- Poston, Tim and Ian Stewart (1978). Catastrophe Theory and its Applications. London, Pitman.
- Prigogine, Ilya and Nicolis, Grégoire (1989). Exploring Complexity : An Introduction, New York: Freeman.
- Prigogine, Ilya and Isabelle Stengers (1984). Order out of Chaos: Man's New Dialogue with Nature. London: Heinemann.
- Thom, René. 1972. Stabilité structurelle et morphogenèse. Paris : Interéditions (English translation 1975)
- Thom, René. 1983. Mathematical Models of Morphogenesis. Horwood (Wiley). New York.
- Zeeman, E. C. 1977. Catastrophe Theory: Selected Papers 1972-1977. Cambridge (Mass.): Addison-Wesley.